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# The Flavor Symmetry <sup>1</sup>

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## ABSTRACT

Assuming that the lepton, quark and Higgs fields belong to the three-dimensional reducible representation of the permutation group  $S_3$ , we suggest a minimal  $S_3$  invariant extension of the standard model. We find that in the leptonic sector, the exact  $S_3 \times Z_2$  symmetry, which allows 6 real independent parameters with two CP-violating phases, is consistent with experimental data and predicts a maximal mixing of the atmospheric neutrinos, and that the third neutrino is the lightest neutrino. With the exact  $S_3$  only, there are 10 real parameters and five phases in the quark sector. A set of values of these parameters that are consistent with experimental observations is given.

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<sup>1</sup>The published version contains wrong statements about CP violations. They are removed in the present version, and the corresponding errors are corrected.

# 1 Introduction

A non-abelian flavor symmetry would explain various phenomena in flavor physics that appear to be independent at present. Moreover, this would provide useful hints about physics beyond the standard model (SM). In this paper we argue that there exists such a symmetry at the Fermi scale. This symmetry is the permutation symmetry  $S_3$ [1, 2], which is the smallest non-abelian symmetry. It is the symmetry of an equilateral triangle, and has a simple geometrical interpretation.

The product groups  $S_3 \times S_3$  and  $S_3 \times S_3 \times S_3$  have been considered by many authors in the past to explain the hierarchical structure of the fermionic matter in the SM[3, 4]. The introduction of the product groups indeed has proven to be successful.<sup>2</sup> However, these symmetries are explicitly and hardly broken at the Fermi scale. If we accept  $S_3$  as a fundamental symmetry in the matter sector of the SM, we are automatically led to extend the Higgs sector of the SM, because the SM contains only one Higgs  $SU(2)_L$  doublet, which can only be an  $S_3$  singlet: Since  $S_3$  has two irreducible representations<sup>3</sup>, singlet and doublet, there is no convincing reason why there should exist only an  $S_3$  singlet Higgs. In fact, along this line of thought, interesting models based on  $S_3$ ,  $S_4$  and also  $A_4$  have been considered [1, 6, 7, 8, 9]. However, the equality of the irreducible representations has not been stressed in these works. The permutation symmetry  $S_3$  means the equality not only of three objects, but also of its irreducible representations. Nevertheless, it allows differences among the generations that are realized in the nature of elementary particles, as we will see.

## 2 A minimal $S_3$ invariant extension of the standard model

Consider a set of three objects,  $(f_1, f_2, f_3)$ , and their six possible permutations. They are the elements of  $S_3$ , which is the discrete non-abelian group with the smallest number of elements. The three-dimensional representation is not an irreducible representation of  $S_3$ . It can be decomposed into the direct sum of two irreducible representations, a doublet  $\mathbf{f}_D$  and a singlet  $\mathbf{f}_S$ , where

$$\mathbf{f}_S = \frac{1}{\sqrt{3}}(f_1 + f_2 + f_3), \quad \mathbf{f}_D^T = \left( \frac{1}{\sqrt{2}}(f_1 - f_2), \frac{1}{\sqrt{6}}(f_1 + f_2 - 2f_3) \right). \quad (1)$$

Two-dimensional matrix representations,  $D_i$ , of  $S_3$  can be obtained from

$$D_+(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad \text{and} \quad D_-(\theta) = \begin{pmatrix} -\cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (2)$$

with  $\theta = 0, \pm 2\pi/3$ , where  $\det D_{\pm} = \pm 1$ . The angles  $\theta$ 's correspond to the symmetry of an equilateral triangle. The tensor product of two doublets,  $\mathbf{p}_D^T = (p_{D1}, p_{D2})$  and  $\mathbf{q}_D = (q_{D1}, q_{D2})$ , contain two singlets,  $\mathbf{r}_S$  and  $\mathbf{r}_{S'}$ , and one doublet,  $\mathbf{r}_D = (r_{D1}, r_{D2})$ , where

$$\mathbf{r}_S = p_{D1}q_{D1} + p_{D2}q_{D2}, \quad \mathbf{r}_{S'} = p_{D1}q_{D2} - p_{D2}q_{D1}, \quad (3)$$

$$\mathbf{r}_D^T = (r_{D1}, r_{D2}) = (p_{D1}q_{D2} + p_{D2}q_{D1}, p_{D1}q_{D1} - p_{D2}q_{D2}). \quad (4)$$

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<sup>2</sup>See, for instance, Ref. [5].

<sup>3</sup>By this we mean that the three-dimensional representation of  $S_3$ , for which the permutations are explicit, can be decomposed into the direct sum of two irreducible representations.

Note that  $\mathbf{r}_{S'}$  is not an  $S_3$  invariant, while  $\mathbf{r}_S$  is.

After the short description of  $S_3$  given above, it is straightforward to extend the SM: In addition to the SM Higgs fields  $H_S$ , we introduce an  $S_3$ -doublet Higgs  $H_D$ . The quark, lepton and Higgs fields are

$$Q^T = (u_L, d_L) \quad , \quad u_R \quad , \quad d_R \quad , \quad L^T = (\nu_L, e_L) \quad , \quad e_R \quad , \quad \nu_R \quad , \quad H \quad (5)$$

with obvious notation. All of these fields have three species, and we assume that each forms a reducible representation  $\mathbf{1}_S + \mathbf{2}$ . The doublets carry capital indices  $I$  and  $J$ , which run from 1 to 2, and the singlets are denoted by  $Q_3, u_{3R}, d_{3R}, L_3 \quad , \quad e_{3R} \quad , \quad \nu_{3R} \quad , \quad H_S$ . Note that the subscript 3 has nothing to do with the third generation. The most general renormalizable Yukawa interactions are given by

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}, \quad (6)$$

where

$$\begin{aligned} \mathcal{L}_{Y_D} = & -Y_1^d \overline{Q}_I H_S d_{IR} - Y_3^d \overline{Q}_3 H_S d_{3R} \\ & -Y_2^d [ \overline{Q}_I \kappa_{IJ} H_1 d_{JR} + \overline{Q}_I \eta_{IJ} H_2 d_{JR} ] \\ & -Y_4^d \overline{Q}_3 H_I d_{IR} - Y_5^d \overline{Q}_I H_I d_{3R} + \text{h.c.}, \end{aligned} \quad (7)$$

$$\begin{aligned} \mathcal{L}_{Y_U} = & -Y_1^u \overline{Q}_I (i\sigma_2) H_S^* u_{IR} - Y_3^u \overline{Q}_3 (i\sigma_2) H_S^* u_{3R} \\ & -Y_2^u [ \overline{Q}_I \kappa_{IJ} (i\sigma_2) H_1^* u_{JR} + \eta \overline{Q}_I \eta_{IJ} (i\sigma_2) H_2^* u_{JR} ] \\ & -Y_4^u \overline{Q}_3 (i\sigma_2) H_I^* u_{IR} - Y_5^u \overline{Q}_I (i\sigma_2) H_I^* u_{3R} + \text{h.c.}, \end{aligned} \quad (8)$$

$$\begin{aligned} \mathcal{L}_{Y_E} = & -Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} \\ & -Y_2^e [ \overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR} ] \\ & -Y_4^e \overline{L}_3 H_I e_{IR} - Y_5^e \overline{L}_I H_I e_{3R} + \text{h.c.}, \end{aligned} \quad (9)$$

$$\begin{aligned} \mathcal{L}_{Y_\nu} = & -Y_1^\nu \overline{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (i\sigma_2) H_S^* \nu_{3R} \\ & -Y_2^\nu [ \overline{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR} ] \\ & -Y_4^\nu \overline{L}_3 (i\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \overline{L}_I (i\sigma_2) H_I^* \nu_{3R} + \text{h.c.}, \end{aligned} \quad (10)$$

and

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (11)$$

Furthermore, we introduce the Majorana mass terms for the right-handed neutrinos

$$\mathcal{L}_M = -M_1 \nu_{IR}^T C \nu_{IR} - M_3 \nu_{3R}^T C \nu_{3R}, \quad (12)$$

where  $C$  is the charge conjugation matrix.

Because of the presence of three Higgs fields, the Higgs potential  $V_H(H_S, H_D)$  is more complicated than that of the SM. But we may assume that all the VEV's are real and that  $\langle H_1 \rangle = \langle H_2 \rangle$ .<sup>4</sup> They also satisfy the constraint  $\langle H_S \rangle^2 + \langle H_1 \rangle^2 + \langle H_2 \rangle^2 \simeq (246 \text{ GeV})^2/2$ . Then

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<sup>4</sup>See, for instance, Ref. [7] in which a potential with three Higgs fields of  $S_3$  is considered.

from the Yukawa interactions (7)–(10) and (12) one derives the mass matrices, which have the general form

$$\mathbf{M} = \begin{pmatrix} m_1 + m_2 & m_2 & m_5 \\ m_2 & m_1 - m_2 & m_5 \\ m_4 & m_4 & m_3 \end{pmatrix}. \quad (13)$$

The Majorana masses for  $\nu_L$  can be obtained from the see-saw mechanism[10], and the corresponding mass matrix is given by  $\mathbf{M}_\nu = \mathbf{M}_{\nu_D} \tilde{\mathbf{M}}^{-1} (\mathbf{M}_{\nu_D})^T$ , where  $\tilde{\mathbf{M}} = \text{diag}(M_1, M_1, M_3)$ . All the entries in the mass matrices can be complex; there is no restriction coming from  $S_3$ . Therefore, there are  $4 \times 5 = 20$  complex parameters in the mass matrices, which should be compared with  $4 \times 9 = 36$  of the SM with the Majorana masses of the left-handed neutrinos. The mass matrices are diagonalized by the unitary matrices as

$$U_{d(u,e)L}^\dagger \mathbf{M}_{d(u,e)} U_{d(u,e)R} = \text{diag}(m_{d(u,e)}, m_{s(c,\mu)}, m_{b(t,\tau)}), \quad (14)$$

$$U_\nu^T \mathbf{M}_\nu U_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \quad (15)$$

The mixing matrices are then defined as <sup>5</sup>

$$V_{CKM} = U_{uL}^\dagger U_{dL}, \quad V_{MNS} = U_{eL}^\dagger U_\nu. \quad (16)$$

### 3 The leptonic sector and $Z_2$ symmetry

To achieve further reduction of the number of parameters, we introduce a  $Z_2$  symmetry. The  $Z_2$  assignment in the leptonic sector is given in the Table I.

**Table I.**  $Z_2$  assignment in the leptonic sector.

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$H_S, \nu_{3R}$	$H_I, L_3, L_I, e_{3R}, e_{IR}, \nu_{IR}$

The  $Z_2$  symmetry forbids certain couplings:

$$Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu = 0. \quad (17)$$

[The  $Z_2$  assignment above is not the unique assignment to achieve (17).] Since  $m_1^e = m_3^e = 0$  due to the  $Z_2$  symmetry, all the phases appearing in (13) can be removed by a redefinition of  $L_I, L_3$  and  $e_{3R}$ . Then, we calculate the unitary matrix  $U_{eL}$  from

$$U_{eL}^\dagger \mathbf{M}_e \mathbf{M}_e^\dagger U_{eL} = \text{diag}(|m_e|^2, |m_\mu|^2, |m_\tau|^2), \quad (18)$$

where

$$\mathbf{M}_e \mathbf{M}_e^\dagger = \begin{pmatrix} 2(m_2^e)^2 + (m_5^e)^2 & (m_5^e)^2 & 2m_2^e m_4^e \\ (m_5^e)^2 & 2(m_2^e)^2 + (m_5^e)^2 & 0 \\ 2m_2^e m_4^e & 0 & 2(m_4^e)^2 \end{pmatrix}. \quad (19)$$

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<sup>5</sup>We denote the physical neutrino masses by  $m_{\nu_i}$ , but  $\nu_{iL}$  are not the mass eigenstates.

All the parameters in (19) are real. The Majorana masses of the right-handed neutrinos  $M_1$  and  $M_3$  in (12) may be complex. They can be made real by a redefinition of the right-handed neutrino fields. Then we redefine  $m_i^\nu$  according to

$$(m_2^\nu) \rightarrow \rho_2^\nu = (m_2^\nu)/\sqrt{M_1}, \quad (m_4^\nu) \rightarrow \rho_4^\nu = (m_4^\nu)/\sqrt{M_1}, \quad (m_3^\nu) \rightarrow \rho_3^\nu = (m_3^\nu)/\sqrt{M_3}. \quad (20)$$

to obtain the Majorana masses of the left-handed neutrinos in the form

$$\mathbf{M}_\nu = \mathbf{M}_{\nu_D} \tilde{\mathbf{M}}^{-1} (\mathbf{M}_{\nu_D})^T = \begin{pmatrix} 2(\rho_2^\nu)^2 & 0 & 2\rho_2^\nu \rho_4^\nu \\ 0 & 2(\rho_2^\nu)^2 & 0 \\ 2\rho_2^\nu \rho_4^\nu & 0 & 2(\rho_4^\nu)^2 + (\rho_3^\nu)^2 \end{pmatrix}. \quad (21)$$

All the phases in (21), except for e.g.  $\rho_3^\nu$ , can be eliminated. However, in the following discussions, we assume that  $(\rho_3^\nu)^2$  is real, so that  $\rho_3^\nu$  is either a real or a purely imaginary number.

Now, consider the limit  $m_4^\nu \rightarrow 0$  in (19). One of the eigenvalues of (19) becomes 0. Therefore, we assume that  $m_e^2 \sim (m_4^\nu)^2$ . In this limit, the other eigenvalues,  $(m_\mu^2, m_\tau^2)$ , and the corresponding eigenvectors,  $\mathbf{v}_\mu$  and  $\mathbf{v}_\tau$ , are given by

$$(m_\mu^2, m_\tau^2) = (2(m_2^\nu)^2, 2(m_2^\nu)^2 + 2(m_5^\nu)^2), \quad (22)$$

$$\mathbf{v}_\mu = (-1/\sqrt{2}, 1/\sqrt{2}), \quad \mathbf{v}_\tau = (1/\sqrt{2}, 1/\sqrt{2}). \quad (23)$$

Therefore,  $U_{eL}$  in this limit becomes

$$U_{eL}^0 = \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 1 & 0 & 0 \end{pmatrix}. \quad (24)$$

The correction to the eigenvalues due to the nonvanishing  $m_5^\nu$  can be computed, and we find

$$m_e^2 = \frac{(m_4^\nu m_5^\nu)^2}{(m_2^\nu)^2 + (m_5^\nu)^2} + O((m_4^\nu)^4), \quad (25)$$

$$m_\mu^2 = 2(m_2^\nu)^2 + (m_4^\nu)^2 + O((m_4^\nu)^4), \quad (26)$$

$$m_\tau^2 = 2[(m_2^\nu)^2 + (m_5^\nu)^2] + \frac{(m_4^\nu m_2^\nu)^2}{(m_2^\nu)^2 + (m_5^\nu)^2} + O((m_4^\nu)^4). \quad (27)$$

For the mass values  $m_e = 0.51$  MeV,  $m_\mu = 105.7$  MeV and  $m_\tau = 1777$  MeV (which correspond to  $m_4^\nu/m_2^\nu = 0.006836$  and  $m_5^\nu/m_2^\nu = 16.78$ ), we obtain

$$U_{eL} \simeq \begin{pmatrix} 3.4 \times 10^{-3} & 1/\sqrt{2} + O(10^{-5}) & 1/\sqrt{2} + O(10^{-10}) \\ -3.4 \times 10^{-3} & -1/\sqrt{2} + O(10^{-5}) & 1/\sqrt{2} + O(10^{-10}) \\ -1 + O(10^{-5}) & 4.8 \times 10^{-3} & O(10^{-5}) \end{pmatrix}. \quad (28)$$

In the neutrino sector, one immediately finds that one of the eigenvalues of  $\mathbf{M}_\nu$  is  $2(\rho_2^\nu)^2$  with the eigenvector  $(0, 1, 0)$ . [Recall that  $(2\rho_3^\nu)^2$  is assumed to be real.] The other eigenvalues  $m_\pm$  are

$$m_\pm = \frac{1}{2}(A \pm [-8(\rho_2^\nu \rho_3^\nu)^2 + A^2]^{1/2}), \quad (29)$$

where  $A = 2(\rho_2^\nu)^2 + (\rho_3^\nu)^2 + 2(\rho_4^\nu)^2$ . Noticing that

$$\tilde{A} = -8(\rho_2^\nu \rho_3^\nu)^2 + A^2 = (-2(\rho_2^\nu)^2 + (\rho_3^\nu)^2)^2 + 4(\rho_4^\nu)^2((\rho_4^\nu)^2 + 2(\rho_2^\nu)^2 + (\rho_3^\nu)^2),$$

we find that  $m_-$  ( $m_+$ ) reaches the maximal (minimal) value,  $2(\rho_2^\nu)^2$ , at  $\tilde{A} = 0$ , if  $(\rho_3^\nu)^2$  is positive. For a positive  $(\rho_3^\nu)^2$ , we can obtain  $\tilde{A} = 0$  if  $2(\rho_2^\nu)^2 = (\rho_3^\nu)^2$  and  $(\rho_4^\nu)^2 = 0$  are satisfied. Therefore,

$$m_- \leq 2(\rho_2^\nu)^2 \leq m_+ \quad (30)$$

should be satisfied if  $(\rho_3^\nu)^2$  is positive. Consequently, if  $\rho_3^\nu$  is real,  $2(\rho_2^\nu)^2$  cannot be identified with  $m_{\nu_3}$ , which comes from the experimental constraint  $|\Delta m_{12}^2| \ll |\Delta m_{23}^2|$ . Thus, we have to identify it with  $m_{\nu_1}$ . Consequently, we arrive at

$$(U_\nu)_{21} = 1, (U_\nu)_{11} = (U_\nu)_{31} = 0, \quad (31)$$

which does not yield the experimentally preferred bi-maximal form of the mixing matrix  $V_{\text{MNS}} = U_{eL}^\dagger U_\nu$  with  $U_{eL}$  given in (28). The bi-maximal form[4] may be obtained if  $U_\nu$  takes the form

$$U_\nu^{\text{max}} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix}. \quad (32)$$

Therefore, to realize the bi-maximal form,  $2(\rho_2^\nu)^2$  has to be the smallest eigenvalue of the mass matrix (21). This, however, is impossible if  $(\rho_3^\nu)^2 \geq 0$ , implying that  $\rho_3^\nu$  has to be purely imaginary (under the assumption that  $(\rho_3^\nu)^2$  is real). Therefore we write  $\rho_3^\nu$  as

$$(\rho_3^\nu)^2 = -|\rho_3^\nu|^2. \quad (33)$$

As a consequence, the mass relation  $2(\rho_2^\nu)^2 < |m_-|, |m_+|$  is realized, so that the third neutrino becomes the lightest neutrino with the mass

$$m_{\nu_3} = 2(\rho_2^\nu)^2. \quad (34)$$

This is one of the important predictions of the  $S_3 \times Z_2$  symmetry, under the assumption that  $(\rho_3^\nu)^2$  is real.

Now consider the limit  $\rho_2^\nu \rightarrow 0$  with the constraint  $(M_\nu)_{33} = 0$ , that is  $2(\rho_4^\nu)^2 - |\rho_3^\nu|^2 = 0$ . Then the eigenvalues are given by

$$\left( m_+ = 2|\rho_4^\nu \rho_2^\nu| + (\rho_2^\nu)^2, m_- = -2|\rho_4^\nu \rho_2^\nu| + (\rho_2^\nu)^2, m_{\nu_3} = 2(\rho_2^\nu)^2 \right), \quad (35)$$

and

$$U_\nu \rightarrow U_\nu^{\text{max}},$$

where  $U_\nu^{\text{max}}$  is given in (32), and  $m_{\nu_1} = |m_-|$  and  $m_{\nu_2} = |m_+|$ . So, the limiting form is exactly the bi-maximal form.

The closed form for  $U_\nu$  is found to be

$$U_\nu = \begin{pmatrix} \sin \hat{\theta} & \cos \hat{\theta} & 0 \\ 0 & 0 & 1 \\ -\cos \hat{\theta} & \sin \hat{\theta} & 0 \end{pmatrix}, \quad (36)$$

with

$$\tan \hat{\theta} = \left( \frac{m_{+(-)} - m_{\nu_3}}{m_{\nu_3} - m_{-(+)}} \right)^{1/2} \quad \text{for } |m_+| > (<) |m_-|, \quad (37)$$

where  $m_\pm$  is given in (29) with the replacement  $(\rho_3^\nu)^2 \rightarrow -|\rho_3^\nu|^2$ , and  $m_{\nu_2} = |m_{+(-)}|$  and  $m_{\nu_1} = |m_{-(+)}|$  for  $|m_+| > (<) |m_-|$ . Then, together with (28), we obtain

$$\begin{aligned} |V_{\text{MNS}}| &= |U_{eL}^\dagger P U_\nu| \\ &\simeq \begin{pmatrix} \cos \theta_{\text{sol}} & \sin \theta_{\text{sol}} & |U_{e3}| \\ \sin \theta_{\text{sol}}/\sqrt{2} & \cos \theta_{\text{sol}}/\sqrt{2} & \sin \theta_{\text{atm}} \\ \sin \theta_{\text{sol}}/\sqrt{2} & \cos \theta_{\text{sol}}/\sqrt{2} & \cos \theta_{\text{atm}} \end{pmatrix}, \end{aligned} \quad (38)$$

where  $P = \text{diag.}(1, 1, \exp i[\arg(Y_4^\nu) - \arg(Y_2^\nu) + \arg(Y_2^e) - \arg(Y_4^e)])$ ,

$$\tan \theta_{\text{atm}} = 1, \quad (39)$$

$$\tan \theta_{\text{sol}} = \tan \hat{\theta}, \quad (40)$$

$$|U_{e3}| = 3.4 \times 10^{-3} + O(m_e m_\mu / m_\tau^2). \quad (41)$$

[Similar, but different predictions have been made in Ref. [11].] The results above (38)-(41) do not change even if the assumption on the reality of  $(\rho_3^\nu)^2$  is removed. In Fig. 1, we plot  $\tan \theta_{\text{sol}}$  as a function of  $x = |\Delta m_{23}^2|/m_{\nu_2}^2$  for  $r = |\Delta m_{12}^2|/|\Delta m_{23}^2| = 0.05$  (dashed), 0.0264 (solid), 0.2 (dot-dashed), where  $|\Delta m_{23}^2| = m_{\nu_2}^2 - m_{\nu_3}^2$  and  $|\Delta m_{12}^2| = m_{\nu_2}^2 - m_{\nu_1}^2$ . Note that in the present model  $m_{\nu_3} < m_{\nu_2}, m_{\nu_1}$ . In Ref. [13] (see the references therein and also Ref. [14]), the experimental data recently obtained in different neutrino experiments, including solar[17], atmospheric[18], accelerator neutrino[19] and reactor[20] experiments are reviewed. It is concluded that

$$\begin{aligned} \tan \theta_{\text{atm}} &= 0.65 - 1.5, \quad \tan \theta_{\text{sol}} = 0.53 - 0.93, \\ |\Delta m_{12}^2|/\text{eV}^2 &= 5.1 \times 10^{-5} - 9.7 \times 10^{-5} \text{ or } 1.2 \times 10^{-4} - 1.9 \times 10^{-4}, \\ |\Delta m_{23}^2|/\text{eV}^2 &= 1.2 \times 10^{-3} - 4.8 \times 10^{-3}, \\ |U_{e3}| &< 0.2. \end{aligned}$$

Comparing Fig. 1, (39) and (41) with the experimental values above, we see that our prediction based on the exact  $S_3 \times Z_2$  symmetry in the leptonic sector is consistent with the most recent experimental data on neutrino oscillations and neutrino masses and mixings.

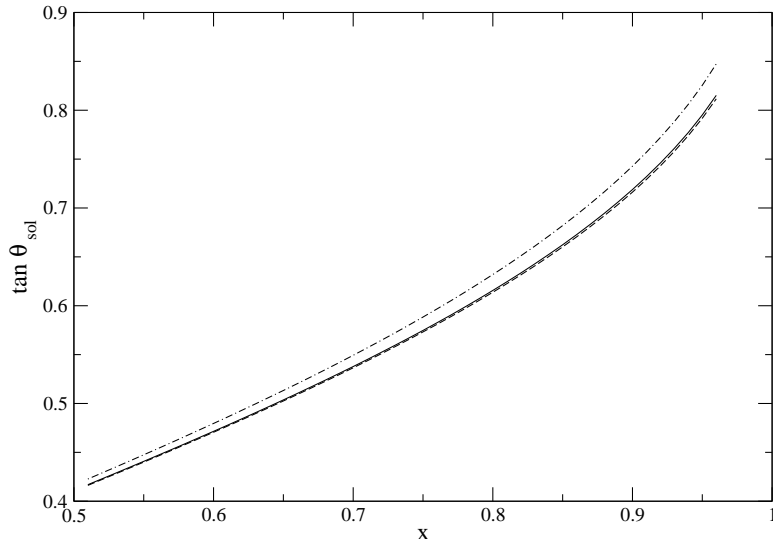


Figure 1:  $\tan \theta_{\text{sol}}$  as a function of  $x = |\Delta m_{23}^2|/m_{\nu_2}^2$  for  $r = |\Delta m_{12}^2|/|\Delta m_{23}^2| = 0.05$  (dashed),  $0.0264$  (solid),  $0.2$  (dot-dashed).

## 4 The hadronic sector

Now we come to the hadronic sector. At the level of the  $S_3$  extension of the SM, the  $Z_2$  assignment in the hadronic sector is independent of that of the leptonic sector. (The  $Z_2$  assignment in the leptonic sector is given in the Table I.) Since the  $Z_2$  symmetry in the leptonic sector seems to be a good symmetry, we assume that it is a good symmetry at a more fundamental level, too. Therefore, we require that the  $Z_2$  symmetry is free from any quantum anomaly.<sup>6</sup> Furthermore, we assume that the quarks and leptons are unified at the fundamental level, which is possible if they have the same  $Z_2$  assignment, implying that all the quarks should have even parity. One can easily convince oneself that under this  $Z_2$  assignment the non-abelian gauge anomalies

$$Z_2[SU(2)_L]^2, \quad Z_2[SU(3)]^2, \quad Z_2[U(1)_Y]^2, \quad [Z_2]^3 \quad (42)$$

cancel at the Fermi scale, because only the right-handed  $S_3$  singlet neutrino among fermions has an odd  $Z_2$  parity.

As in the case of the leptonic sector, the  $Z_2$  symmetry forbids the Yukawa couplings  $Y_1^{u,d}$  and  $Y_3^{u,d}$  [see (17)]. We proceed to consider the generation structure in the hadronic sector under the assumption that  $Z_2$  is explicitly broken in this sector. Since all the  $S_3$  invariant Yukawa couplings are now allowed, the mass matrices for the quarks take the general form (13), in which all the entries can be complex. One can easily see that all the phases, except for those of  $m_1^{u,d}$  and  $m_3^{u,d}$ , can be removed through an appropriate redefinition of the quark fields. Here, we assume that only  $m_3^d$  is a complex number. The unitary matrices  $U_{uL}$  and  $U_{dL}$  can be obtained from

$$U_{u(d)L}^\dagger \mathbf{M}_{u(d)} \mathbf{M}_{u(d)}^\dagger U_{u(d)L} = \text{diag}(|m_{u(d)}|^2, |m_{c(s)}|^2, |m_{t(b)}|^2), \quad (43)$$

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<sup>6</sup>Anomalies of discrete symmetries are discussed in Ref. [12].



where

$$\mathbf{M}_{u(d)} = \begin{pmatrix} m_1^{u(d)} + m_2^{u(d)} & m_2^{u(d)} & m_5^{u(d)} \\ m_2^{u(d)} & m_1^{u(d)} - m_2^{u(d)} & m_5^{u(d)} \\ m_4^{u(d)} & m_4^{u(d)} & m_3^{u(d)} \end{pmatrix}. \quad (44)$$

To diagonalize the mass matrices, we start by observing that realistic mass hierarchies can be achieved in the following way. In the limit  $m_{4,5} \rightarrow 0$ , they become block-diagonal, and  $m_3$  becomes an eigenvalue whose eigenvector is  $(0, 0, 1)$ . The  $2 \times 2$  blocks, which are of a semi-democratic type, can be simply diagonalized. One finds easily that one of the eigenvalues can become 0 if  $m_1^2 - 2m_2^2 = 0$  is satisfied. So, the gross structure of realistic mass matrices can be obtained if  $m_3^{u,d} \sim O(m_{t,b})$  and  $m_{1,2}^{u,d} \sim O(m_{c,s})$  (to realize realistic mass hierarchies), and the non-diagonal elements  $m_4^{u,d}$  and  $m_5^{u,d}$ , along with  $m_{1,2}^{u,d}$ , can produce a realistic mixing among the quarks. There are 10 real parameters and one phase (under the assumption that only  $m_3^d$  is complex) in order to produce six quark masses, three mixing angles and one  $CP$ -violating phase. It is certainly desirable to investigate the complete parameter space of the model to understand its phenomenology and to make predictions, if any can be obtained. However, this is a quite complex problem, and will go beyond the scope of the present paper. Here we would like to give one set of parameters that are consistent with the experimental values given by the Particle Data Group[15]. We find that the set of dimensionless parameter values

$$\begin{aligned} m_1^u/m_0^u &= -0.000293, \quad m_2^u/m_0^u = -0.00028, \quad m_3^u/m_0^u = 1, \\ m_4^u/m_0^u &= 0.031, \quad m_5^u/m_0^u = 0.0386, \\ m_1^d/m_0^d &= 0.0004, \quad m_2^d/m_0^d = 0.00275, \quad m_3^d/m_0^d = 1 + 1.2I, \\ m_4^d/m_0^d &= 0.283, \quad m_5^d/m_0^d = 0.058 \end{aligned} \quad (45)$$

yields the mass hierarchies

$$\begin{aligned} m_u/m_t &= 1.33 \times 10^{-5}, \quad m_c/m_t = 2.99 \times 10^{-3}, \\ m_d/m_b &= 1.31 \times 10^{-3}, \quad m_s/m_b = 1.17 \times 10^{-2}, \end{aligned} \quad (46)$$

where  $m_0^u = m_3^u$  and  $m_0^d = \text{Re}(m_3^d)$ , and the mixing matrix becomes

$$\begin{aligned} V_{\text{CKM}} &= U_{uL}^\dagger U_{dL} \\ &= \begin{pmatrix} 0.968 + 0.117I & 0.198 + 0.0974I & -0.00253 - 0.00354I \\ -0.198 + 0.0969I & 0.968 - 0.115I & -0.0222 - 0.0376I \\ 0.00211 + 0.00648I & 0.0179 - 0.0395I & 0.999 - 0.00206I \end{pmatrix}. \end{aligned} \quad (47)$$

The magnitudes of the elements are given by

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.975 & 0.221 & 0.00435 \\ 0.221 & 0.974 & 0.0437 \\ 0.00682 & 0.0434 & 0.999 \end{pmatrix}, \quad (48)$$

which should be compared with the experimental values[15]

$$|V_{\text{CKM}}^{\text{exp}}| = \begin{pmatrix} 0.9741 \text{ to } 0.9756 & 0.219 \text{ to } 0.226 & 0.0025 \text{ to } 0.0048 \\ 0.219 \text{ to } 0.226 & 0.9732 \text{ to } 0.9748 & 0.038 \text{ to } 0.044 \\ 0.004 \text{ to } 0.014 & 0.037 \text{ to } 0.044 & 0.9990 \text{ to } 0.9993 \end{pmatrix}. \quad (49)$$

Note that the mixing matrix (47) is not in the standard parametrization. So, we give the invariant measure of  $CP$ -violations[16]

$$J = \text{Im} [(V_{\text{CKM}})_{11}(V_{\text{CKM}})_{22}(V_{\text{CKM}}^*)_{12}(V_{\text{CKM}}^*)_{21}] = 2.5 \times 10^{-5} \quad (50)$$

for the choice (46), which is slightly larger than the experimental value  $(3.0 \pm 0.3) \times 10^{-5}$  (see [15] and also Ref. [21]). The angles of the unitarity triangle for  $V_{\text{CKM}}$  (47) are given by

$$\phi_1 \simeq 22^\circ, \quad \phi_3 \simeq 38^\circ, \quad (51)$$

where the experimental values are  $\phi_1 = 24^\circ \pm 4^\circ$  and  $\phi_3 = 59^\circ \pm 13^\circ$  [15]. The normalization masses  $m_0^u$  and  $m_0^d$  are fixed at

$$m_0^u = 174 \text{ GeV}, \quad m_0^d = 1.8 \text{ GeV} \quad (52)$$

for  $m_t = 174 \text{ GeV}$  and  $m_b = 3 \text{ GeV}$ , yielding  $m_u \simeq 2.3 \text{ MeV}$ ,  $m_c \simeq 0.52 \text{ GeV}$ ,  $m_d \simeq 3.9 \text{ MeV}$  and  $m_s = 0.035 \text{ GeV}$ . Although these values cannot be directly compared with the running masses, because our calculation is at the tree level, it is nevertheless worthwhile to observe how close they are to [5]

$$\begin{aligned} m_u(M_Z) &= 0.9 - 2.9 \text{ MeV}, \quad m_d(M_Z) = 1.8 - 5.3 \text{ MeV}, \\ m_c(M_Z) &= 0.53 - 0.68 \text{ GeV}, \quad m_s(M_Z) = 0.035 - 0.100 \text{ GeV}, \\ m_t(M_Z) &= 168 - 180 \text{ GeV}, \quad m_b(M_Z) = 2.8 - 3.0 \text{ GeV}. \end{aligned} \quad (53)$$

## 5 Flavor changing neutral currents (FCNCs)

In models with more than one Higgs  $SU(2)_L$  doublet, as in the case of the present model, tree-level FCNCs exist in the Higgs sector. We therefore calculate the flavor changing Yukawa couplings to the neutral Higgs fields,  $H_S^0$  and  $H_I^0$  ( $I = 1, 2$ ), where  $H_S^0$  and  $H_I^0$  stand for the neutral Higgs fields of the  $S_3$ -singlet  $H_S$  and the  $S_3$ -doublet  $H_I$ , respectively. The actual values of these couplings depend on the VEV's of the Higgs fields, and hence on the Higgs potential, which we do not consider in the present paper. Since we only would like to estimate the size of the tree-level FCNCs here, we simply assume that

$$\langle H_S^0 \rangle = \langle H_1^0 \rangle = \langle H_2^0 \rangle \simeq 246/\sqrt{6} \text{ GeV} \simeq 142/\sqrt{2} \text{ GeV}. \quad (54)$$

Then, the flavor changing Yukawa couplings can explicitly be calculated, because we know the explicit values of the unitary matrices  $U$ 's defined in (14):

$$\begin{aligned} \mathcal{L}_{\text{FCNC}} &= \left( \overline{E}_{aL} Y_{ab}^{ES} E_{bR} + \overline{U}_{aL} Y_{ab}^{US} U_{bR} + \overline{D}_{aL} Y_{ab}^{DS} D_{bR} \right) H_S^0 + \text{h.c.} \\ &+ \left( \overline{E}_{aL} Y_{ab}^{E1} E_{bR} + \overline{U}_{aL} Y_{ab}^{U1} U_{bR} + \overline{D}_{aL} Y_{ab}^{D1} D_{bR} \right) H_1^0 + \text{h.c.} \\ &+ \left( \overline{E}_{aL} Y_{ab}^{E2} E_{bR} + \overline{U}_{aL} Y_{ab}^{U2} U_{bR} + \overline{D}_{aL} Y_{ab}^{D2} D_{bR} \right) H_2^0 + \text{h.c.} \end{aligned} \quad (55)$$

Here the matrices  $E$ 's,  $U$ 's and  $D$ 's stand for the mass eigenstates, and

$$Y^{E1} \simeq \begin{pmatrix} -10^{-5} & 2.6 \times 10^{-6} & -4.2 \times 10^{-5} \\ -1.1 \times 10^{-3} & 5.3 \times 10^{-4} & -8.8 \times 10^{-3} \\ -1.2 \times 10^{-8} & 5.3 \times 10^{-4} & -8.8 \times 10^{-3} \end{pmatrix}, \quad (56)$$

$$Y^{E2} \simeq \begin{pmatrix} 5.1 \times 10^{-6} & -2.6 \times 10^{-6} & 4.2 \times 10^{-5} \\ 1.1 \times 10^{-3} & 5.3 \times 10^{-4} & 8.8 \times 10^{-3} \\ 1.2 \times 10^{-8} & -5.3 \times 10^{-4} & -8.8 \times 10^{-3} \end{pmatrix}, \quad (57)$$

$$Y^{US} \simeq \begin{pmatrix} -4.7 \times 10^{-4} & 3.8 \times 10^{-4} & -7.1 \times 10^{-3} \\ -3.8 \times 10^{-4} & -3.6 \times 10^{-3} & 7.5 \times 10^{-2} \\ 8.8 \times 10^{-3} & 9.4 \times 10^{-2} & -1.7 \end{pmatrix}, \quad (58)$$

$$Y^{U1} \simeq \begin{pmatrix} 8.3 \times 10^{-4} & 1.6 \times 10^{-3} & -3.4 \times 10^{-2} \\ -1.6 \times 10^{-3} & 4.9 \times 10^{-3} & -4.1 \times 10^{-2} \\ 4.3 \times 10^{-2} & -5.1 \times 10^{-2} & -4.2 \times 10^{-3} \end{pmatrix}, \quad (59)$$

$$Y^{U2} \simeq \begin{pmatrix} -3.3 \times 10^{-4} & -1.9 \times 10^{-3} & 4.1 \times 10^{-2} \\ 1.9 \times 10^{-3} & 3.8 \times 10^{-3} & -3.4 \times 10^{-2} \\ -5.1 \times 10^{-2} & -4.2 \times 10^{-2} & -4.2 \times 10^{-3} \end{pmatrix}, \quad (60)$$

$$Y^{DS} \simeq \begin{pmatrix} (1.4 + 0.44I) \times 10^{-5} & (5.6 + 0.38I) \times 10^{-5} & -(1.6 + 1.6I) \times 10^{-4} \\ (5.5 + 0.38I) \times 10^{-5} & (2.8 - 2.1I) \times 10^{-4} & -(1.4 + 0.16I) \times 10^{-3} \\ -(7.8 + 8.0I) \times 10^{-4} & -(7.0 + 0.82I) \times 10^{-3} & (1.8 + 2.1I) \times 10^{-2} \end{pmatrix}, \quad (61)$$

$$Y^{D1} \simeq \begin{pmatrix} -(1.1 - 0.17I) \times 10^{-4} & -(1.6 - 1.3I) \times 10^{-4} & (8.0 - 0.033I) \times 10^{-4} \\ -(1.6 - 1.3I) \times 10^{-4} & -(2.4 - 1.7I) \times 10^{-4} & (6.1 + 1.6I) \times 10^{-4} \\ 4.0 \times 10^{-3} & (3.1 + 0.82I) \times 10^{-3} & (6.1 + 7.3I) \times 10^{-4} \end{pmatrix}, \quad (62)$$

$$Y^{D2} \simeq \begin{pmatrix} (6.2 - 0.71I) \times 10^{-5} & (1.1 - 1.4I) \times 10^{-4} & -(6.4 - 1.6I) \times 10^{-4} \\ (1.0 - 1.3I) \times 10^{-4} & -(3.0 - 2.8I) \times 10^{-4} & (7.5 - 0.054I) \times 10^{-4} \\ -(3.2 + 0.80I) \times 10^{-3} & (3.9 - 0.001I) \times 10^{-3} & (6.1 + 7.3I) \times 10^{-4} \end{pmatrix}. \quad (63)$$

All the non-diagonal elements are responsible for tree-level FCNC processes. The amplitude of the flavor violating process  $\mu^- \rightarrow e^+ e^- e^-$ , for instance, is proportional to  $(Y^{E1})_{11}(Y^{E1})_{21} \simeq 10^{-8}$ . Then, we find that its branching ratio is estimated to be

$$B(\mu \rightarrow 3e) \sim 10^{-15} (M_W/M_H)^4 < 10^{-12}, \quad (64)$$

where  $M_W$  and  $M_H$  are the  $W$  boson mass and Higgs mass, respectively, and the value  $10^{-12}$  is the experimental upper bound. Similarly, we obtain

$$B(\tau \rightarrow 3\mu) \sim 10^{-10} (M_W/M_H)^4 < 10^{-6}, \quad (65)$$

$$B(K_L^0 \rightarrow 2e) \sim 10^{-16} (M_W/M_H)^4 < 10^{-12}, \quad (66)$$

$$B(B_S^0 \rightarrow 2\mu) \sim 10^{-7} (M_W/M_H)^4 < 10^{-6}. \quad (67)$$

Note that because of the three Higgs fields, the imaginary parts of the  $Y$ 's contribute to  $CP$ -violating amplitudes, which are not taken into account by the phase of the mixing matrix  $V_{\text{CKM}}$ . Therefore, the independent phases in the mass matrices (13) can be, in principle, measured. A complete analysis of this problem will go beyond the scope of the present paper, and we would like to leave this problem to a future work.

## 6 Conclusion

$S_3$  is a non-abelian permutation group with the smallest number of elements. The symmetry  $S_{3L} \times S_{3R}$  has been considered by many authors[3, 4, 5] in the past to explain the hierarchal structure of the generations in the SM.  $S_{3L} \times S_{3R}$  is, however, explicitly and hardly broken at the Fermi scale. In the present paper we considered its diagonal subgroup, while extending the concept of flavor and generation to the Higgs sector. Once this is done, there is no reason that there should exist only an  $S_3$  singlet Higgs, and so we introduced three  $SU(2)_L$  Higgs doublet fields. The minimal  $S_3$  extension of the SM allows a definite structure of the Yukawa couplings, and we studied its consequences, in particular the mass of the quarks and leptons, and their mixings. Although similar ideas have been proposed previously [1, 6, 7, 8, 9], none of the existing treatments is identical to ours: The main differences are the inclusion of  $(S_3 - \text{doublet})^3$  couplings and the arrangement of the  $S_3$  representations. We found that in the leptonic sector, an additional discrete symmetry  $Z_2$  can be consistently introduced. The theoretical predictions obtained from the  $S_3 \times Z_2$  symmetry are consistent with the experimental observations made to this time. As for the quark sector, we simply assumed that  $Z_2$  is explicitly broken and analyzed the quark mass matrices that result only from  $S_3$ . We found that they are consistent with experiments. From these studies we hypothesize that the flavor symmetry, which is exact at the Fermi scale, is the permutation symmetry  $S_3$ . This flavor symmetry, together with the electroweak gauge symmetry, is only spontaneously broken. The analysis of the mass matrices in the hadronic sector that we performed in the present paper, is by no means complete, because we gave only one set of consistent parameter values. Also, the analysis of the FCNCs in the Higgs sector is not complete. A more complete study will be published elsewhere.

Supersymmetrization of the model is straightforward. It will simplify the Higgs sector drastically, and, moreover, the squared soft scalar masses will enjoy a certain degree of degeneracy, thanks to  $S_3$ , thereby softening the supersymmetric flavor problem.

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